

REAL-TIME LINEAR QUADRATIC CONTROL USING DIGITAL SIGNAL PROCESSOR

T. SLAVOV¹, L. MOLLOV¹, P. PETKOV¹

ABSTRACT. In this paper, a system for real-time Linear Quadratic Control (LQR) of multi-variable analogue plant is presented. The system consists of Spectrum Digital eZdspTMF28335 development kit with built in Texas Instruments TMS320F28335 Digital Signal Processor (DSP), Pulse Width Modulation (PWM) to analogue signal converter and voltage divider. An appropriate software in MATLAB[®]/Simulink[®] environment is developed which is embedded in DSP by using the Simulink Coder[®]. The LQR controller and state observer designed provide acceptable system performance. Experimental and simulation results are presented which confirm that the control system achieves the prescribed performance.

Keywords: LQR regulator, state observer, digital signal controller, real-time control.

AMS Subject Classification: 93C35, 93C41, 93C62, 93C80.

1. INTRODUCTION

The satisfaction of the requirements to the performance of modern control systems makes necessary plant modeling that takes into account the interaction between input/output channels and usage of sophisticated control algorithms based on state space models. The availability of high performance microprocessor techniques allows such systems to be implemented successfully in practice. To shorten the development time it is desirable to test the control algorithm under conditions close to the real ones. For this aim it is appropriate to design a laboratory setup, which consists of a functionally equivalent physical model and a digital controller implementing the control algorithm. The development of such a system allows to solve a wide set of research problems such as simulation, investigation of system sensitivity to plant parameter variations, study of system performance in presence of disturbances and noises, prediction of system behavior in operation conditions that are not allowable for the real plant and so on.

In this paper we present a laboratory system for real-time Linear Quadratic Control of multi-variable analogue 4th order two input/two output plant under incomplete information about the state vector. The system consists of Spectrum Digital eZdspTMF28335 development kit with built in Texas Instruments TMS320F28335 Digital Signal Processor, Pulse Width Modulation to analogue signal converter, voltage divider and a test analogue plant. An appropriate software in MATLAB[®]/Simulink[®] environment is developed which is embedded in DSP by using the Simulink Coder[®]. The usage of this new technology facilitates the implementation of complex algorithms for control and observation of multivariable plants. The LQR controller with integral components and state observer designed provides acceptable system performance in presence of output disturbances. Frequency domain analysis of closed-loop performance is done. Experimental and simulation results are presented which confirm that the control system achieves the prescribed performance.

¹Department of Systems and Control, Technical University of Sofia, Bulgaria,
e-mail: php@tu-sofia.bg

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2. AN ALGORITHM FOR INTEGRAL LQR CONTROL UNDER INCOMPLETE STATE INFORMATION

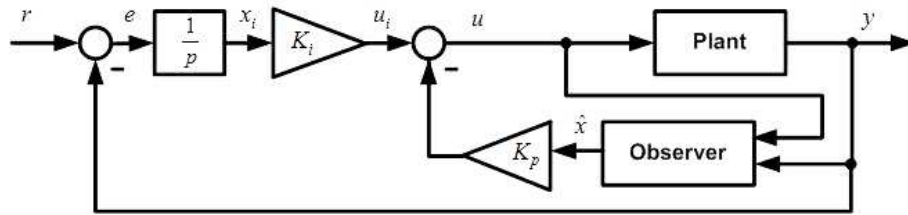


Figure 1. Block-diagram of the closed-loop system

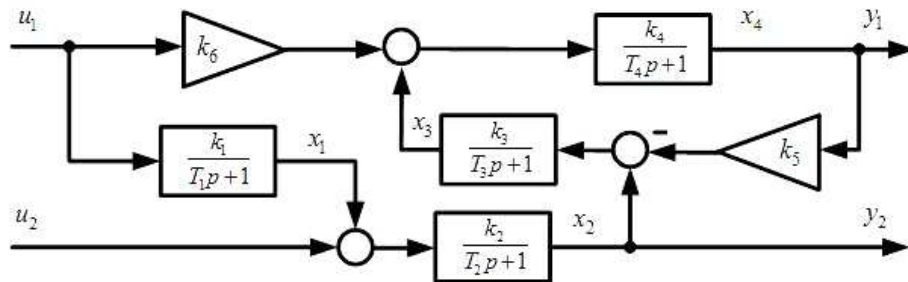


Figure 2. Block-diagram of the plant

The block-diagrams of the closed-loop system and the plant are shown in Figures 1 and 2, respectively.

After introducing the state vector $[x_1 \ x_2 \ x_3 \ x_4]^T$ the plant description is obtained as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{1}$$

where

$$A = \begin{bmatrix} -\frac{1}{T_1} & 0 & 0 & 0 \\ \frac{k_2}{T_2} & -\frac{1}{T_2} & 0 & 0 \\ 0 & \frac{k_3}{T_3} & -\frac{1}{T_3} & 0 \\ 0 & 0 & \frac{k_4}{T_4} & -\frac{1}{T_3} \end{bmatrix}, B = \begin{bmatrix} \frac{k_1}{T_1} & 0 \\ 0 & \frac{k_2}{T_2} \\ 0 & 0 \\ \frac{k_4 k_6}{T_4} & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}, D = I_{2 \times 2},$$

k_1, k_2, \dots, k_6 are coefficients of proportionality, T_1, T_2, \dots, T_4 are time constants, $u(t) = [u_1(t), u_2(t)]$ is the input vector and $y(t) = [y_1(t), y_2(t)]$ is the output vector.

The LQR regulator with integrating action for the plant (1) under full information about the state is described by the equation

$$u(t) = K_i x_i(t) - K_r x(t), \tag{2}$$

where

$$x_i(t) = \int_0^t e(\tau) d\tau, \tag{3}$$

K_i, K_r are the regulator matrices and $e(t) = r(t) - y(t)$ is the output error.

Let us introduce the extended state vector

$$\bar{x}(t) = [x_i(t), x(t)]^T. \tag{4}$$

After differentiation of (3) in respect to time and taking into account (2) and (4), one obtains the equation for control system with full information about the state

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{B}u + \bar{J}r, \\ u &= -\bar{K}_p\bar{x}\end{aligned}\quad (5)$$

where

$$\bar{A} = \begin{bmatrix} 0_{2 \times 2} & -C \\ 0_{4 \times 2} & A \end{bmatrix}, \bar{B} = \begin{bmatrix} 0_{2 \times 2} \\ B \end{bmatrix}, \bar{J} = \begin{bmatrix} I_{2 \times 2} \\ 0_{4 \times 2} \end{bmatrix}, \bar{K}_r = [K_i \ K_r].$$

For the regulator design, the quadratic performance index

$$J = \int_0^{\infty} (\bar{x}^T Q \bar{x} + u^T R u) dt, \quad (6)$$

is used where

$$Q = \begin{bmatrix} 30 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

are weighting matrices chosen as to ensure the desired performance of the closed-loop system transient responses. The regulator matrix \bar{K}_r which minimizes (6) is determined by the expression [1]

$$K_p = R^{-1} \bar{B}^T P, \quad (7)$$

where P is the positive definite solution of the matrix Riccati equation

$$\bar{A}^T P + P \bar{A} - P \bar{B} R^{-1} \bar{B}^T P + Q. \quad (8)$$

After solving (8) and substituting the solution into (7), one obtains for the regulator matrix

$$\bar{K}_p = \begin{bmatrix} -54.7685 & -0.2609 & 1.2909 & 0.2089 & -0.2272 & 10.6379 \\ 0.6391 & -22.3592 & 0.7302 & 10.0837 & 0.7920 & -0.2136 \end{bmatrix}.$$

The control of the system, shown in Figure 2, is determined by the equation

$$u(t) = K_i x_i(t) - K_r \hat{x}(t), \quad (9)$$

where $\hat{x}(t)$ is the estimate of $x(t)$. This estimate is obtained by the state observer

$$\dot{\hat{x}}(t) = (A - K_H C) \hat{x} + B u(t) + K_H y(t), \quad (10)$$

where K_H is the matrix gain. It is determined by the desired observer poles

$$\lambda_1 = -145.3212, \lambda_2 = -48.3581, \lambda_3 = -4.6169, \lambda_4 = -1.9507,$$

which are the eigenvalues of the matrix $A - K_H C$.

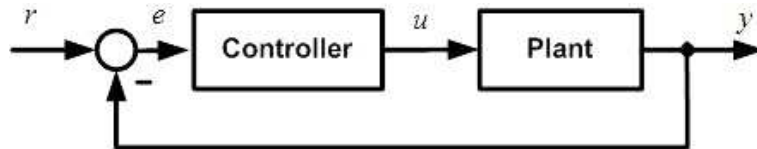


Figure 3. Equivalent standard block-diagram of the control system

With the aim to use the standard functions of Robust Control Toolbox of MATLAB[®] for frequency domain analysis, the system in Figure 1 is transformed in the form of system with one-degree-of-freedom controller, as shown in Figure 3.

The block “Plant” in Figure 3 is described by equations (1) and the block “Regulator” is described by

$$\begin{aligned} \dot{x}_p(t) &= A_p x_p(t) + B_p e(t), \\ u(t) &= C_p x_p(t) + D_p e(t), \end{aligned} \tag{11}$$

where

$$A_p = \begin{bmatrix} 0_{2 \times 2} & 0_{2 \times 4} & 0_{2 \times 4} \\ BK_i & A - B_p K_p & BK_p \\ 0_{4 \times 2} & 0_{4 \times 4} & A - K_H C \end{bmatrix}, B_p = \begin{bmatrix} I_2 \\ 0_{4 \times 2} \\ 0_{4 \times 2} \end{bmatrix}, C_p^T = \begin{bmatrix} K_i \\ -K_p \\ K_p \end{bmatrix}, D_p = 0_{2 \times 2}.$$

FREQUENCY DOMAIN ANALYSIS OF THE SYSTEM

The equivalent system in Figure 3 and the derived description (11) allow to analyze the closed-loop system in the frequency domain using the standard functions of Robust Control Toolbox.

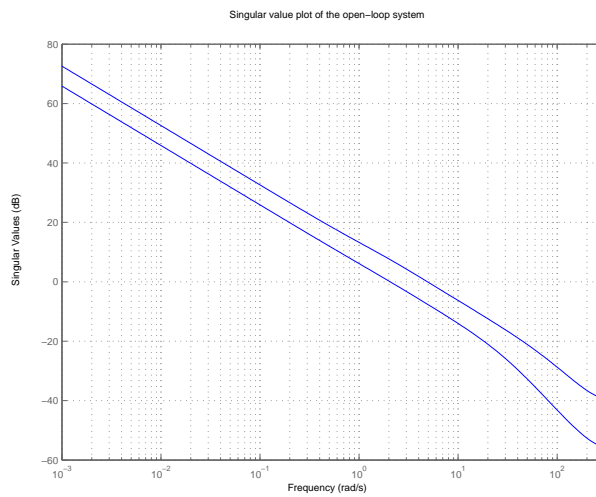


Figure 4. Singular values of the open-loop system

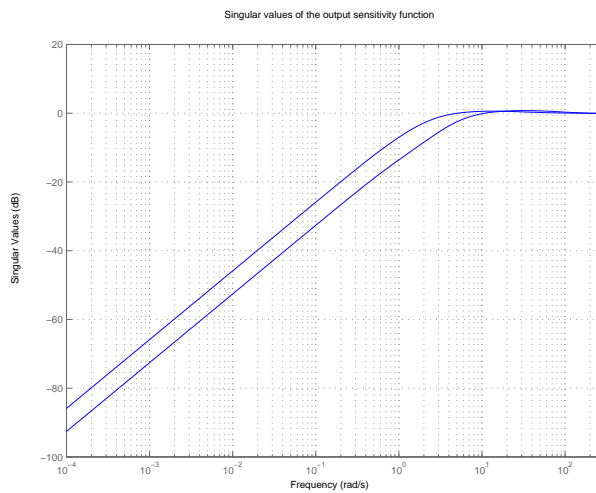


Figure 5. Singular values of the output sensitivity function

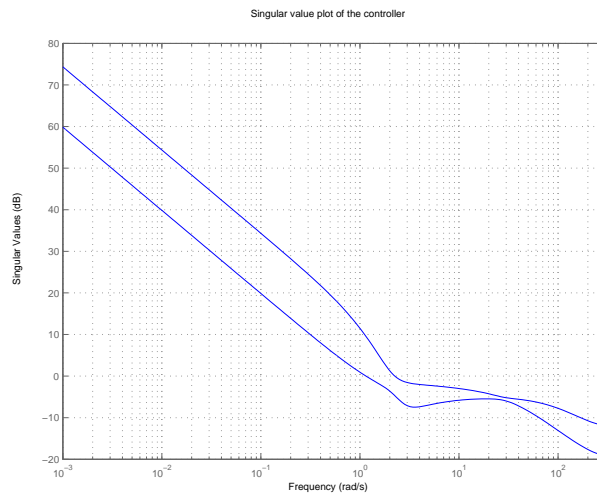


Figure 6. Regulator singular values

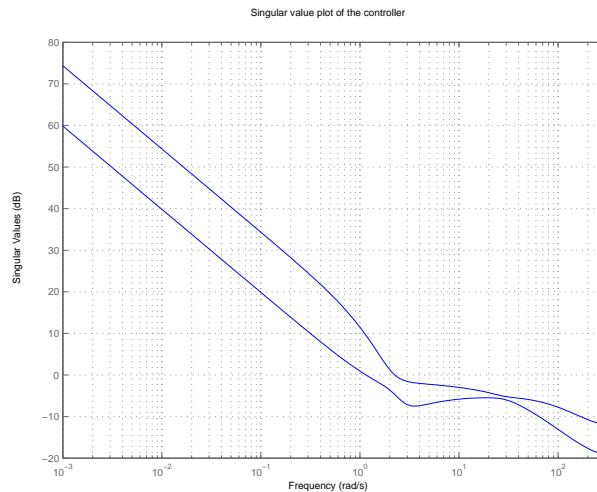


Figure 7. Singular values of reference-to-control transfer function matrix

In Figures 4 – 7 we show the singular value plots of the discrete-time open-loop system, output sensitivity function, regulator transfer function matrix and reference-to-control transfer function matrix. It is seen from the figures that the system attenuates well low frequency output disturbances (for instance, a sinusoidal disturbance with frequency 0.02 rad/s will be attenuated 100 times) and there are no peaks of the output sensitivity function singular value plot in the high frequency range. Also, the regulator suppress well the high frequency measurement noise.

3. IMPLEMENTATION OF LQR WITH INTEGRAL CONTROL BY DSP

A photo of the experimental setup is shown in Figure 8 and the block-diagram of developed control system with DSP is shown in Figure 9.

MIMO plant (1) is modeled on an analog modeling system, consisting of a bus on which functional blocks are operating independently. Bus is powered by a DC power supply. The linear range of input-output signals is $\pm 15V$. The main blocks are adder, differentiator, inverter, integrator aperiodic unit and gain unit. The analog system makes it easy to model plants with a time constant to 1 s and gain factor up to 10.

The block “Voltage divider” is a specially designed dual channel board, which converts linearly two input analog signals with range $-5 \div 5V$ into two analog output signals with range $0 \div 3V$.

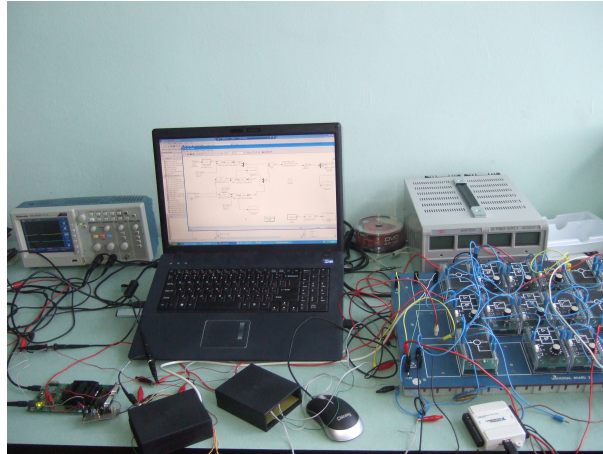


Figure 8. Photo of the experimental setup

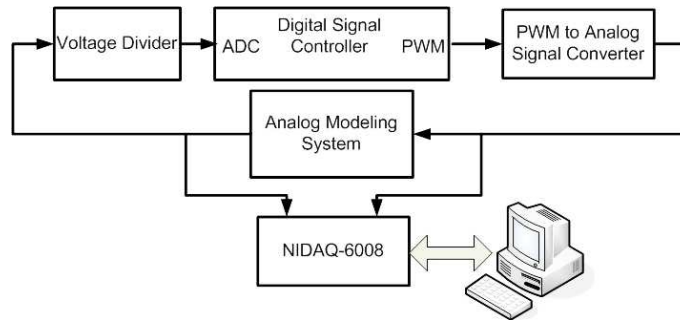


Figure 9. Block-diagram of the control system with DSP

The block “Converter PWM / Analog signal” is a specially designed dual channel board which converts the two input PWM signals with range $0 \div 3V$ to two analog output signals with range $-5 \div 5V$.

The block “NIDAQ-6008” is a specialized module for data collection NIDAQ-6008 of National Instruments, whose characteristics are described in detail in [2]. A specially developed software provides the user interface, connection and exchange data with a standard PC in real time.

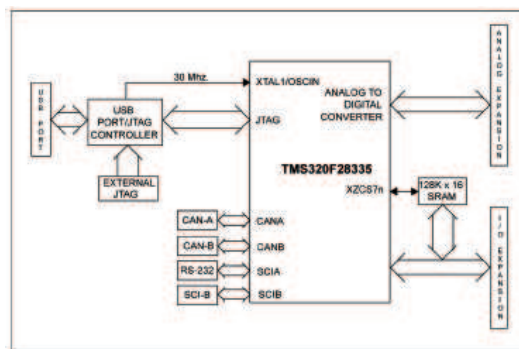


Figure 10. Block-diagram of eZdsp[®]F28335

Block ”Digital Signal Controller” is the Spectrum Digital eZdspTMF28335 development board with an integrated Digital Signal Processor Texas Instruments TMS320F28335. Block-diagram of the development board is shown in Figure 10 [3].

The DSP has on-chip 32-bit floating point unit. Its operating speed is 150MHz and it has 68K bytes on-chip RAM, 256K bytes off-chip SRAM memory, 512K bytes on-chip Flash memory. On-chip 12 bit Analog to Digital (A/D) converter with 16 input channels and pulse width modulator with up to 18 output channels are also integrated. Available are the communication protocols RS-232, CAN2.0, and SPI. Software features are MATLAB[®]/Simulink[®] and TI Code Composer Studio[™] Integrated Development Environment, Version 3.3.

In this work a technology to generate and embed the code using the Simulink Coder[®] has been implemented. Simulink Coder[®] offers native support for compilation and execution of the generated code to the environments Eclipse[®] IDE, Microsoft[®] Visual C++[®] IDE, Linux[®] OS and Windows[®] OS[4]. The main advantages of this technology are the highly shortened time to translate the control algorithm in the working simulation environment to the real working environments with physical plants, reducing the overall time for application testing and verification of the developed algorithm and the relatively easy implementation of complex control algorithms.

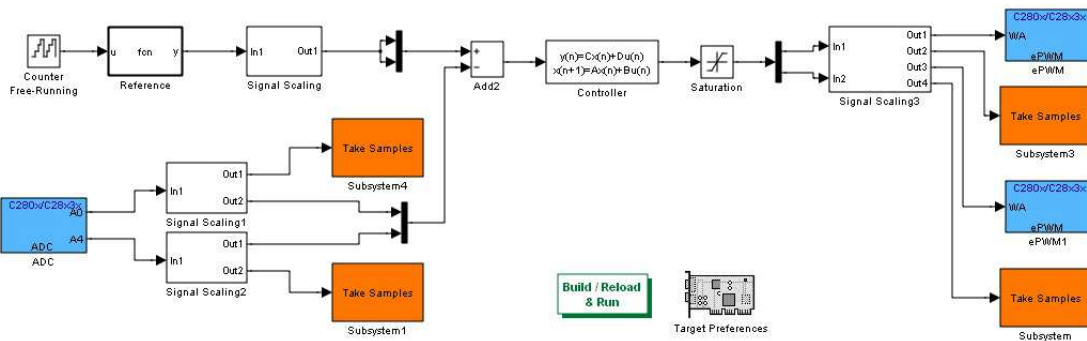


Figure 11. Block-diagram for DSP code generation

The software developed is shown in Figure 11.

The diagram in Figure 11 includes the blocks “ADC”, “ePWM”, “To RTDX” and “Target Preferences” which are taken from the specialized library for Texas Instruments C2000 of Simulink[®].

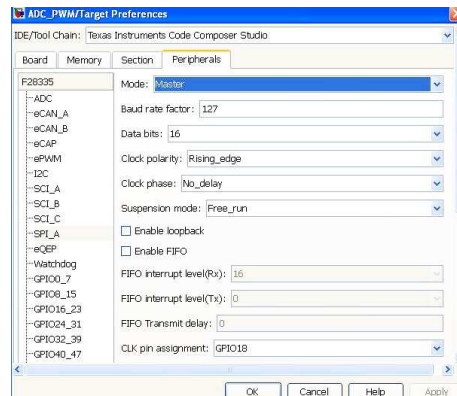


Figure 12. Interface window of “Target Preferences” block

Through the block “Target Preferences”, whose interface window is shown in Figure 12, one can configure specific model of the development board, appropriate model of the DSP and their ways of connecting, operating frequency of the DSP, specific peculiarities of the periphery.

Simulink Coder[®] (previously Real-Time Workshop), Embedded IDE Link, and Simulink[®] use information from “Target Preferences” to properly simulate the model and to generate executable code for the particular work environment.

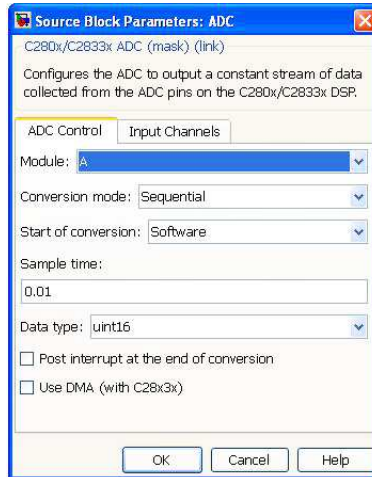


Figure 13. Interface window of ADC block

The “ADC” block, whose interface is shown in Figure 13, is used to configure the analogue-to-digital converter. It is 12-bit with 16 channels, has a minimum sampling rate 80 ns and input voltage $0 \div 3V$. This block can be used as two separate modules of 8 channels (two schemes are available for sample and hold signals) or as a cascade connection of 16 channels. There are two approaches to conversion – sequential and simultaneous, i.e., transformation in both modules simultaneously. There are three different ways to start the conversion: 1) software - converted values are updated every time; 2) ePWM - managed by an event connected with the pulse width modulator; 3) XINT2 ADC - managed by an external signal. The output data type after conversion can be double, single, int8, uint8, int16, uint16, int32 and uint32.

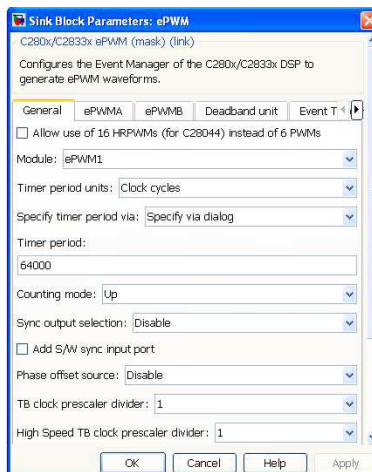


Figure 14. Interface window of ePWM block

The block “ePWM”, whose interface is shown in Figure 14, configures the Event Manager to generate ePWM waveforms. This block allows to perform the following settings: which of the various ePWM is active, which are the units of the timer period – seconds or clock cycles (this is better method because of fewer calculations and reduced rounding error), size of the period, mode of counting - up, down or up and down.

The block “To RTDX” enables to visualize and transfer data in real time from DSP to MATLAB[®]/Simulink[®] environment, by developing a software in MATLAB[®]. Thus one can see immeasurable variables from the executable code on the DSP.

4. EXPERIMENTAL RESULTS

The performance of the developed system for Linear Quadratic Control under incomplete information about the state of an analogue model with digital signal controller is confirmed by experimental investigations. Several experiments are done with variable reference and sinusoidal disturbance signals. A comparison is shown between the experimental results and those obtained by control system simulation.

A discrete-time analogue of the regulator (11) is implemented by the digital signal controller. The description (11) is discretized with a sampling period of $T_0 = 0.01$ s by a method taking into account the zero-order hold. This allows to obtain satisfactory results in the control algorithm implementation by the digital signal processor.

EXPERIMENT WITH VARIABLE REFERENCE

An experiment is done in which the reference varies every 10 s at three levels: -1V, 0V and 1V.

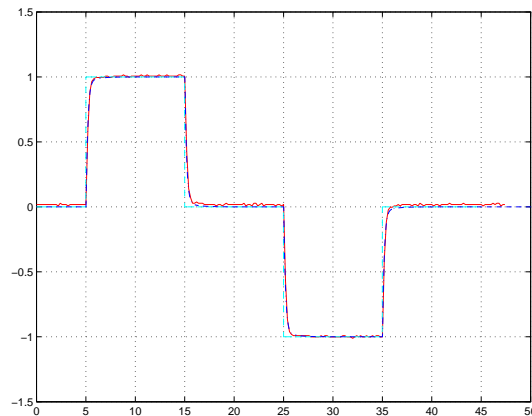


Figure 15. First output signal

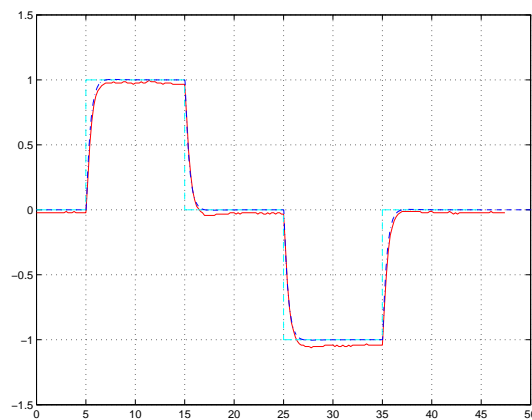


Figure 16. Second output signal

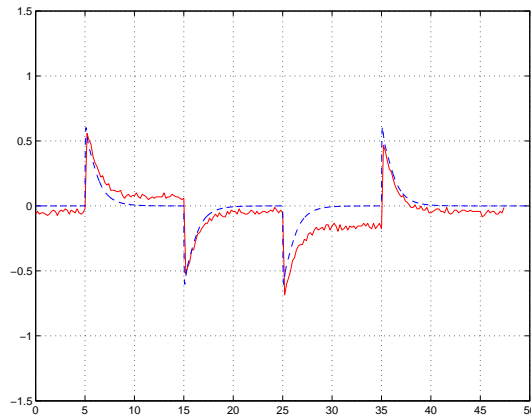


Figure 17. First control signal

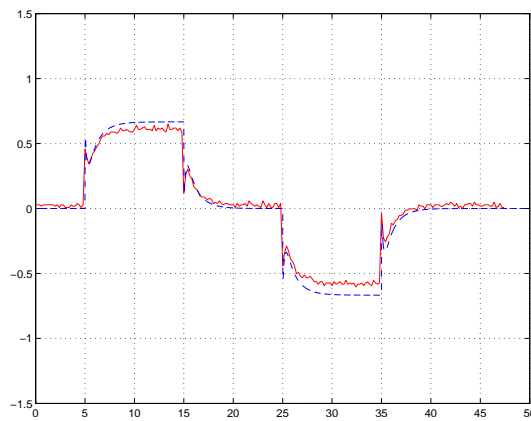


Figure 18. Second control signal

The output signals (here and further on all signals are measured in volts) from simulation and experiments are shown in Figures 15 – 18 (the experimental results are shown in red). It is seen from the Figures displaying the output signals that the experimental signals coincide very well with the simulation signals. The Figures displaying control signals show a small constant deviation of the experimental signals from the simulation signals particularly expressed at low control level. This is due, on one hand, to the zero shift as a result of voltage divider and PWM to analog signal converter biases, and, on the other hand, to the implementation error of the plant gain coefficient on the analogue modeling system.

EXPERIMENT WITH SINUSOIDAL DISTURBANCE

An experiment is done in which a sinusoidal disturbance with magnitude 1V and frequency 1 rad/s is added to the first output of the system at the 5th second.

The output and control signals from experiments and simulation are shown in Figures 19 – 22. After the initial deviation at the 5th second the first output quickly decreases and oscillations with magnitude 0.12V are settled, i.e., the disturbance is attenuated approximately 10 times. The steady-state oscillations around the equilibrium point of the second output are negligible. The Figures displaying control signals show a good coincidence between experimental and simulation results.

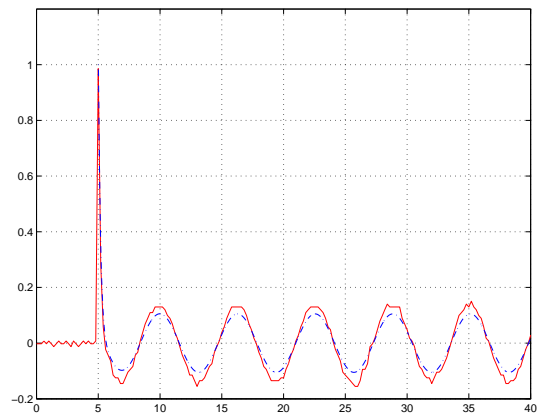


Figure 19. First output signal

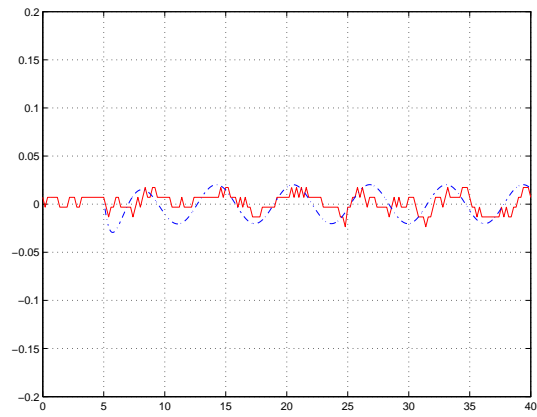


Figure 20. Second output signal

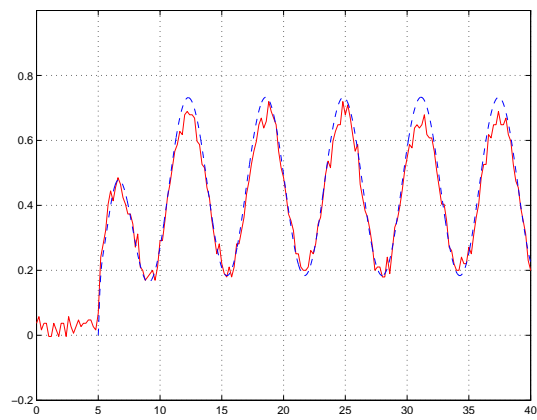


Figure 21. First control signal

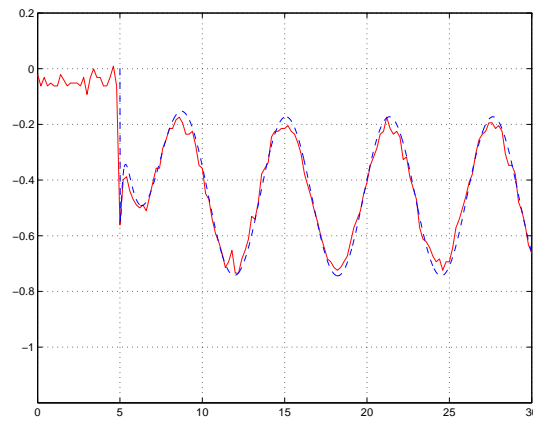


Figure 22. Second control signal

5. CONCLUSION

The paper presents a real-time system for Linear Quadratic Control of fourth order plant under incomplete information about the state. A specialized software for automatic generation of the control code is developed that facilitates the implementation of complex algorithms for control and observation of multivariable plants. The experimental results confirm the performance of the closed-loop system with digital signal controller implementing the linear quadratic control law. The control code may be easily modified for control of other plants.

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Tsonyo Slavov graduated from the Department of Systems and Control of the Technical University of Sofia in 2002. He received his Ph.D degree in control engineering from the Technical University of Sofia in 2007. Presently, he is an Associated Professor in the Department of Systems and Control at the Technical University of Sofia.



Lyuben Mollov graduated from the Department of Systems and Control of the Technical University of Sofia in 2008. Presently, he is a Ph.D. student in the same department.



Petko Petkov received M.S. and Ph.D. degrees in control engineering from the Technical University of Sofia in 1971 and 1979. Since 1995, he has been a Professor of Control Theory in the Department of Systems and Control at the Technical University of Sofia. He is coauthoring *Computational Methods for Linear Control Systems* (Prentice Hall, Hemel Hempstead, U.K., 1991), *Perturbation Theory for Matrix Equations* (North-Holland, Amsterdam, 2003), and *Robust Control Design with MATLAB* (Springer, 2005, 2012).